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HS/XII/A. Sc. Com/M/24

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MATHEMATICS

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

General Instructions :

- (i) All questions are compulsory.
- (ii) This question paper contains 36 questions divided into four Sections—A, B, C and D. Section—A comprises of 20 questions of 1 mark each, Section—B comprises of 6 questions of 2 marks each, Section—C comprises of 6 questions of 4 marks each and Section—D comprises of 4 questions of 6 marks each.
- (iii) There is no overall choice. However, internal choice has been provided in 7 questions of Section—A, 3 questions of Section—B, 5 questions of Section—C and 2 questions of Section—D. You have to attempt only one of the alternatives in all such questions.
- (iv) Use of calculator is not permitted.

(2)

SECTION—A

1. Find the principal value of $\sin^{-1} \frac{1}{2}$. 1

2. Differentiate w.r.t. x : 1

$$y = \log(\log x), \quad x > 1$$

3. Construct a 2×2 matrix $A = [a_{ij}]$, where $a_{ij} = \frac{(i - 2j)^2}{2}$. 1

Or

Find the values of x and y , if

$$\begin{array}{rcl} 3x - y & = & y - 1 \\ 2x - y & = & 3 - 4 \end{array} \quad \begin{array}{l} 1 \\ 2 \end{array} \quad \text{1}$$

4. Find the unit vector in the direction of the sum of the vectors $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + \hat{j} + 3\hat{k}$. 1

5. Find the Cartesian equation of the line which passes through the point $(-2, 4, -5)$ and parallel to the line given by

$$\frac{x + 3}{3} = \frac{y - 4}{5} = \frac{z + 8}{6} \quad \text{1}$$

(3)

6. Integrate the function $\sin x \sin(\cos x)$. 1

Or

Evaluate the definite integral $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$. 1

7. Find $\frac{dy}{dx}$ for the following : 1

$$2x - 3y = \sin y$$

8. Evaluate : 1

$$\int_{-\pi/2}^{\pi/2} \sin^7 x \, dx$$

9. Find the direction cosines of the vector $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$. 1

Or

Find the scalar and vector components of the vector with initial point (2, 1) and terminal point (5, 7). 1

10. Find the value of k , if the function given by

$$f(x) = \begin{cases} kx^2, & x < 1 \\ 4, & x \geq 1 \end{cases}$$

is continuous at $x = 1$. 1

(4)

11. Differentiate $\log (\cos e^x)$ w.r.t. x . 1

Or

- Find $\frac{dy}{dx}$, if $x = a \cos \theta$ and $y = a \sin \theta$. 1

12. The radius of a circle is increasing at the rate of 0.7 cm/s.
What is the rate of increase in its circumference? 1

13. Find the second-order derivative of $\log x$. 1

14. Examine the continuity of the function $f(x) = 5x^3$
at $x = 5$. 1

15. Evaluate : 1

$$\frac{dx}{\sqrt{9 - 25x^2}}$$

Or

- Integrate w.r.t. x : 1

$$\frac{1}{x - x \log x}$$

(5)

- 16.** Verify that $y = e^x - 1$ is a solution of the differential equation $y' - y = 0$. 1

Or

Find the general solution of the differential equation

$$\frac{dy}{dx} = \frac{1}{1 - x^2} y^2. \quad 1$$

Choose the correct answer :

- 17.** Let R be the relation in the set \mathbb{N} given by $R = \{(a, b) : a \leq b, b \leq 6\}$. Then

(a) $(2, 4) \in R$

(b) $(3, 8) \in R$

(c) $(6, 8) \in R$

(d) $(8, 7) \in R$ 1

- 18.** The corner points of the feasible region determined by a system of linear constraints are $(0, 0)$, $(0, 40)$, $(20, 40)$, $(60, 20)$ and $(60, 0)$. The maximum value of $Z = 4x + 3y$ is

(a) 120

(b) 240

(c) 300

(d) 200 1

(6)

19. The matrix $\begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$ is a

- (a) diagonal matrix
- (b) row matrix
- (c) column matrix
- (d) unit matrix

1

Or

The square matrices A and B will be inverse of each other, if

- (a) $AB = O, BA = I$
- (b) $AB = BA = O$
- (c) $AB = BA = I$
- (d) $AB = I, BA = O$

1

20. The value of $(\hat{i} \times \hat{j}) \times \hat{k} + (\hat{j} \times \hat{k}) \times \hat{i}$ is

- (a) 1
- (b) 2
- (c) 0
- (d) -1

1

(7)

SECTION—B

- 21.** Show that the function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = \frac{1}{2x}$ is onto. Here, \mathbb{R} is the set of non-zero real numbers. 2

Or

- Let $A = \mathbb{R} \setminus \{3\}$ and $B = \mathbb{R} \setminus \{1\}$. Consider the function $f : A \rightarrow B$ defined by $f(x) = \frac{x-2}{x-3}$. Show that f is one-one. 2

- 22.** Prove that the function $f(x) = \log x$ is strictly increasing on the interval $(0, \infty)$. 2

- 23.** Evaluate : 2

$$\frac{dx}{(x-1)(x-2)}$$

Or

- Find $\frac{dy}{dx}$, if $xy = y^2 \tan x = y$. 2

- 24.** If $y = 5 \cos x + 3 \sin x$, then prove that

$$\frac{d^2y}{dx^2} + y = 0$$
 2

Or

- Find $\frac{dy}{dx}$, if $x = a(\sin \theta)$ and $y = a(1 - \cos \theta)$. 2

- 25.** Given that the events A and B are such that $P(A) = \frac{1}{2}$, $P(B) = p$ and $P(A \cap B) = \frac{3}{5}$. Find p , if A and B are independent. 2
- 26.** Using determinants, find the value of k so that the points $(3, -2)$, $(k, 2)$ and $(8, 8)$ lie on the same line. 2

SECTION—C

- 27.** Using integrals, find the area enclosed by the circle $x^2 + y^2 = 4$. 4

Or

Evaluate : 4

$$\int \frac{2x - 1}{\sqrt{x^2 - 4x + 3}} dx$$

- 28.** Differentiate $(\log x)^{\cos x}$ w.r.t. x . 4

Or

Using the properties of definite integral, evaluate

$$\int_0^{\pi/2} (2 \log \sin x - \log \sin 2x) dx$$
 4

(9)

- 29.** Show that the relation R on \mathbb{Z} given by $R = \{(x, y) : x - y \text{ is divisible by } 2\}$ is an equivalence relation. 4

Or

Let A and B be two sets. Show that $f : A \rightarrow B \rightarrow A$ such that $f(a, b) = (b, a)$ is one-one and onto. 4

- 30.** Solve the following LPP graphically : 4

Minimize $Z = 200x + 500y$
subject to

$$\begin{aligned}x + 2y &= 10 \\3x + 4y &= 24 \\x, y &\geq 0\end{aligned}$$

- 31.** Find the angle between the two lines whose equations are

$$\frac{x}{3} - \frac{y}{5} = \frac{z}{4} \quad \text{and} \quad \frac{x}{1} - \frac{y}{1} = \frac{z}{2} \quad 4$$

Or

Find θ and ϕ , if $(2\hat{i} - 6\hat{j} + 27\hat{k}) \cdot (\hat{i} + \hat{j} + \hat{k}) = \vec{0}$. 4

- 32.** Find the intervals in which the function f given by $f(x) = 2x^3 - 3x^2 - 36x + 7$ is—

(a) strictly increasing;

(b) strictly decreasing. 4

(10)

Or

If $y = e^{a \cos^{-1} x}$, $-1 \leq x \leq 1$, then show that

$$(1 - x^2)y_2 - xy_1 - a^2y = 0 \quad 4$$

SECTION—D

33. Solve the following system of linear equations : 6

$$\begin{array}{rclcl} x & + & y & + & z & = & 6 \\ x & + & y & + & z & = & 2 \\ 2x & + & y & + & z & = & 1 \end{array}$$

34. Find the shortest distance between the lines l_1 and l_2 , whose vector equations are

$$\begin{aligned} \vec{r} &= (\hat{i} + \hat{j}) + \lambda(2\hat{i} + \hat{j} + \hat{k}) \\ \vec{r} &= (2\hat{i} + \hat{j} + \hat{k}) + \mu(3\hat{i} + 5\hat{j} + 2\hat{k}) \end{aligned} \quad 6$$

35. Show that a closed right circular cylinder of given total surface area and maximum volume is such that its height is equal to its diameter. 6

Or

Solve the linear differential equation

$$\cos^2 x \frac{dy}{dx} - y \tan x = 0 \quad x = \frac{\pi}{4}$$

given that $y = 1$, when $x = 0$. 6

36. State Bayes' theorem and use it to solve the following :
2+4=6

In answering a question on a multiple choice test, a student either knows the answer or guesses. Let $3/4$ be the probability that he knows the answer and $1/4$ be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability $1/4$, what is the probability that the student knows the answer given that he answered it correctly?

Or

A random variable X has the following probability distribution :

X	0	1	2	3	4	5	6	7
$P(X)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2$

Determine the following : 6

- (i) k
- (ii) $P(X \leq 3)$
- (iii) $P(X \leq 6)$
- (iv) $P(0 \leq X \leq 3)$
